LaTeX source file for: Riemann Zeta Function Project

```
\usepackage[utf8]{inputenc}
\usepackage[english]{babel}
\usepackage{amsmath}
\usepackage{amsfonts}
\usepackage{amssymb}
\usepackage{fourier}
\usepackage{hyperref}
\usepackage{graphicx}
\usepackage[utf8]{inputenc}
\author{Jake Warde}

\begin{document}

\title {Riemann Zeta Function: Recreated in LaTeX from Wikipedia Article
    }
\maketitle

\begin {abstract}
I have recreated the article mentioned above to learn more about
    composing mathematics articles using LaTeX. And what a fascinating
    subject! I certainly don't understand much of it but admire minds
    that can. Here is the original article:\url {https://en.wikipedia.
    org/wiki/Riemann_zeta_function}
\end {abstract}


\section {Introduction}
The Riemann zeta function or Euler-Riemann zeta function, $\zeta(s)$, is
     a function of a complex variable s that analytically continues the
     sum of the infinite series
\begin{equation}
\zeta(s)- {\sum_{i=1}^{\infty}}\\\;\frac{1} {n^s}
\end{equation}
which converges when the real part of s is greater than 1. More general
    representations of $\zeta(s)$ for all s are given below.
The Riemann zeta function plays a pivotal role in analytic number theory
     and has applications in physics, probability
theory, and applied statistics.

This function, as a function of a real argument, was introduced and
    studied by Leonhard Euler in the first half of the
eighteenth century without using complex analysis, which was not
    available at that time. Bernhard Riemann in his article
"On the Number of Primes Less Than a Given Magnitude" published in 1859
    extended the Euler definition to a complex
variable, proved its meromorphic continuation and functional equation
    and established a relation between its zeros and
the distribution of prime numbers.

The values of the Riemann zeta function at even positive integers were
    computed by Euler. The first of them, $\zeta(s)$ provides a
    solution to the Basel problem. In 1979 Ap ry proved the
    irrationality of $\zeta(s)$. The values at negative integer points,
     also found by Euler, are rational numbers and play an important
    role in the theory of modular forms. Many
generalizations of the Riemann zeta function, such as Dirichlet series,
    Dirichlet L-functions and L-functions, are known.

\begin{equation}
\zeta(s)\:= {\sum_{i=1}^{\infty}}\:\frac{1}{n^s}\:= \:\frac{1}{1^s}\:+ \
    frac{1}{2^s}\:+\:\frac{1}{3^s\:}+ \dotsi \hspace{.5cm}\sigma\:=\:\
    Re(s)> 1.
\end{equation}
It can also be defined by the integral
\begin{equation}
\zeta(s)\:=\:\frac{1}{\Gamma(s)}\:\int_{0}^{\infty}\;\frac{x^s-1}{e^x
    -1}\,dx
```

```
\end{equation}
where $\Gamma(s)$ is the gamma function.
The Riemann zeta function is defined as the analytic continuation of the function defined for $\sigma > 1$ by the sum of the
preceding series.
Leonhard Euler considered the above series in 1740 for positive integer values of s, and later Chebyshev extended the
definition to real $s > 1$.
The above series is a prototypical Dirichlet series that converges absolutely to an analytic function for s such that $\sigma > 1$
and diverges for all other values of s. Riemann showed that the function defined by the series on the half-plane of
convergence can be continued analytically to all complex values $s /ne 1$. For $s = 1$ the series is the harmonic series which
diverges to $+^\infty$, and

\begin{equation}
\lim_{x \to 1}(s-1)\;(\zeta(s))=1
\end{equation}
Thus the Riemann zeta function is a meromorphic function on the whole complex s-plane, which is holomorphic everywhere except for a simple pole at $s=1$ with residue 1.\vspace{.2cm}

\begin{figure}[hbtp]
 \caption{Riemann zeta function for real s > 1}
 \centering
 \includegraphics [scale=.6]{Mplwp_riemann_zeta_real_positive}
 \end{figure}

 For any positive even integer 2n:
 \begin{equation}
 \zeta\,(2n)=\:\frac{-n^n^+^1\;B_2_n(2\pi)^2^n}{2(2n)!}
 \end{equation}

where $B2n$ is a Bernoulli number.
\vspace{.2cm}
For negative integers, one has
\begin{equation}
\zeta(-n)=-\frac{B_n_-_1}{n+1}
\end{equation}

for $n \geq 1$, so in particular $\zeta $B_m = 0$ for all odd $m$ other than $1$. For odd positive integers, no such simple expression is known, although these values are thought to be related to the algebraic K-theory
of the integers; see Special values of L-functions.

Via analytic continuation, one can show that

\begin{equation}
\zeta(-1)= -\frac{1}{12}
\end{equation}
gives a way to assign a finite result to the divergent series $1 + 2 + 3 + 4 + \ldots$ which can be useful in certain contexts such as string theory.

\begin{equation}
\zeta(0)=-\frac{1}{2}
\end{equation}
\begin{equation}
\zeta(\frac{1}{2})\approx -1.4603545)
\end{equation}
This is employed in calculating of kinetic boundary layer problems of linear kinetic equations.

\begin {equation}
\zeta (1)= 1+\frac{1}{2} + \frac{1}{3}+ \ldots =  \infty
\end{equation}
```

```
if we approach from numbers larger than 1. Then this is the harmonic series. But its Cauchy principal value

\begin {equation}
\lim_{x \to 0}\frac {\zeta (1+\varepsilon) + \zeta (1-\varepsilon)} {2}
\end{equation}
exists which is the Euler Mascheroni constant $\gamma=.05772....$
\\

\textbf{This ends the sample document. Go to \url {https://en.wikipedia.org/wiki/Riemann_zeta_function} to read the entire Wikipedia document.}

\end{document}
```